# Quantum Monte Carlo, keeping up with the HPC Evolution

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# Acknowledgements

## QMCPACK developers\*

- Kenneth P. Esler (Stoneridge)
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#### **QMC** Endstation

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<sup>\*</sup>http://qmcpack.cmscc.org

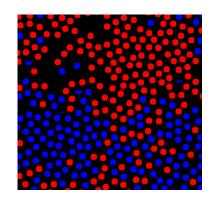
### **Outline**

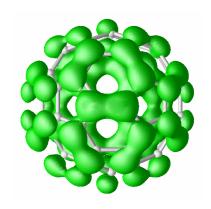
- Quantum Monte Carlo Methods: accurate, robust and efficient solution for electronic structure calculations, especially for correlated systems
- QMC on clusters of multi-core and GPUs
  - OpenMP/MPI hybrid
  - CUDA/MPI hybrid
- Prospect of QMC algorithms on hybrid architectures
- Conclusions

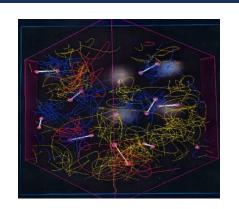


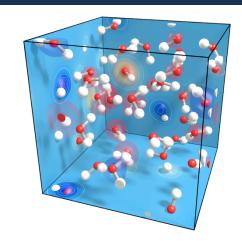


# Quest for Accurate Quantum Simulations: harnessing computing power









- Hard-core bosons on a CDC 6600 (1974)
- Electronic and structure properties of carbon/silicon clusters on HP 9000/715 cluster and Cray Y-MP (1995)
- Coupled Electron-Ion Monte Carlo simulations of dense hydrogen on Linux Clusters (2000)
- Diffusion Monte Carlo simulations of liquid water on multicore SMP clusters (2009)





# QMC advantages: accuracy and scalability

- Applicable to a wide range of problems
  - Any boundary conditions: molecular and solid-state systems
  - Dimensionality: 1D, 2D, and 3D
  - Representation: atomistic to model Hamiltonians
- Scale with a few powers in system size:  $O(N^3)-O(N^4)$ 
  - Routine calculations of 100s-1000s electrons
- Ample opportunities of parallelism

QMC has enabled accurate predictions of correlated electronic systems: plasmas to molecules to solids; insulators to highly correlated metals

- Fundamental High-Pressure Calibration from All-Electron
  Quantum Monte Carlo Calculations, Esler et al, PRL (2010)
- Evidence for a first-order liquid-to-liquid transition in highpressure hydrogen, Morales et al, PNAS (2010)





# QMCPACK: QMC for HPC

- Implements essential QMC algorithms and best practices developed over 20yrs+
- Designed for large-scale QMC simulations of molecules, solids and nanostructures on massively parallel machine
  - (OpenMP,CUDA)/MPI Hybrid parallelization
  - Object-oriented and generic programming in C++
- Apply software engineering
  - Reusable and extensible solution for new development
  - Standard open-source libraries and utilities for development, compilation and execution
  - Portable and scalable I/O with XML/HDF5 http://qmcpack.cmscc.org

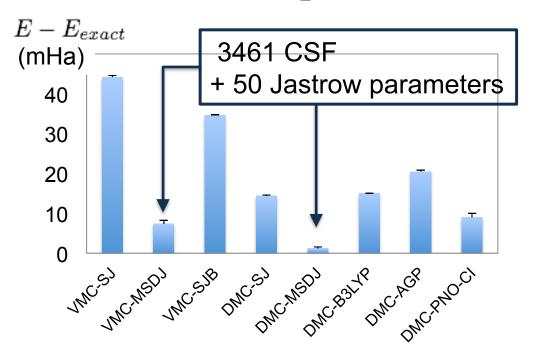




# More recent QMC development\*

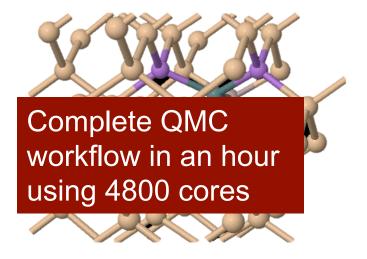
- Efficient and scalable QMC algorithms
- Fast algorithm for multi-determinant evaluation
- Improved energy minimization in VMC and DMC

#### Energy of H<sub>2</sub>O



Formation energy of a native defect in Si

$$E_f$$
= 3.07 (11) eV

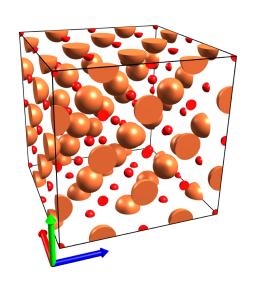


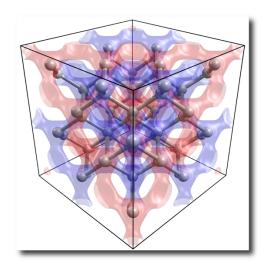
\* By QMCPACK developers

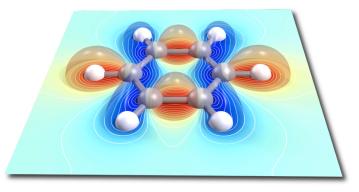


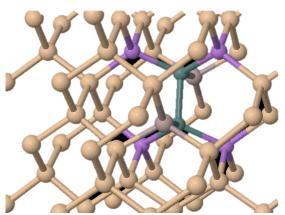


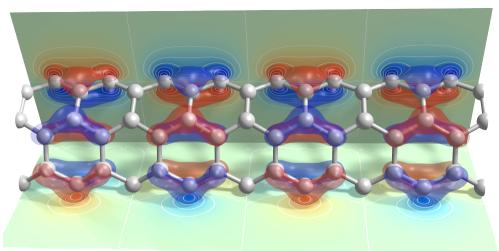
# QMC in Action















# QMC keeping up with HPC evolution

- Increasing accuracy, computational complexity and problem size of QMC simulations with HPC evolution
  - Model Hamiltonian in 70s, e.g., hard-sphere and LJ potential
  - Homogeneous electron gas in 80s, seminal work by Ceperley and Alder laid the foundation of DFT
  - Atoms, molecules and bulk
  - Recently, routine QMC simulations of 1000s of electrons including disordered solids
- Shorter time-to-solution = More Science
- Can QMC continue?





# High-performance computing in 2010s

- Petaflop machines have been around, e.g. Jaguar (OLCF)
- Sustainable petaflop machines are coming, e.g., Blue Waters at NCSA in 2011

#### Clusters of Shared-memory Processors (SMP)

- Hierarchical memory and communication
- Fast interconnects & various inter-node topology
- Increasing number of cores per SMP node
  - 8-32 cores are common; more is expected.
- Fixed memory per core but more aggregated memory per node
- SIMD units: SSE on x86 and VSX on IBM Power 7(P7)
- Large number of threads: simultaneous multi-threading (a.k.a. hyperthreading), e.g., 128 threads on IBM P7 32-core node





### **Basics of QMC**

For N-electron system

$$\{\mathbf{R}\}=(\mathbf{r}_1,\cdots,\mathbf{r}_N)$$

Many-body Hamiltonian

$$\hat{H} = \sum_{i} \frac{1}{2m_e} \nabla^2 + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} + \sum_{i} V_{ext}(\mathbf{r}_i)$$

Find the solution  $\hat{H}|\Psi>=E_0|\Psi>$  &  $\langle\Psi|\hat{H}|\Psi\rangle=E_0$ 

Many-body *trial* wavefunction  $\Psi_T(\mathbf{R})$ 

$$E_T = \frac{\int d^{3N} \mathbf{R} \ \Psi_T^*(\mathbf{R}) \hat{H} \Psi_T(\mathbf{R})}{\int d^{3N} \mathbf{R} \ |\Psi_T(\mathbf{R})|^2}, \quad E_T \ge E_0$$



**QMC** 

$$\langle E_T \rangle = \frac{\sum_i^M w(\mathbf{R}_i) E_L(\mathbf{R}_i)}{\sum_i^M w(\mathbf{R}_i)}, \quad E_L = \frac{\hat{H}\Psi_T(\mathbf{R})}{\Psi_T(\mathbf{R})}$$





# **Essentials of QMC**

#### Note that

$$E_T = \langle E_T \rangle |_{M \to \infty}, \quad E_0 \leftarrow E_T |_{\Psi_T \to \Psi}$$

#### QMC methods employ

- $\Psi_T(\mathbf{R})$  , compact, fast to compute, and accurate
- Efficient stochastic sampling to generate large M
- Variational Monte Carlo (VMC) Variational parameters  $E_{VMC} = \min_{\alpha} \langle \Psi_T(\mathbf{R}; \alpha) | \hat{H} | \Psi_T(\mathbf{R}; \alpha) \rangle$   $|\Psi_T|^2$
- Diffusion Monte Carlo (DMC)

$$E_{DMC} = \langle \Phi_0 | \hat{H} | \Psi_T \rangle, \quad \Phi_0 = \lim_{\beta \to \infty} \exp^{-\beta \hat{H}} \Psi_T \qquad \Phi_0 \Psi_T$$





# Efficiency of QMC

QMC employs sampling to obtain

$$< E_T> = \frac{\sum_i^M w(\mathbf{R}_i) E_L(\mathbf{R}_i)}{\sum_i^M w(\mathbf{R}_i)}, \quad E_L = \frac{\hat{H} \Psi_T(\mathbf{R})}{\Psi_T(\mathbf{R})}$$
 with an error bar  $\delta = \frac{\sigma}{\sqrt{M}}, \quad \sigma^2 = < E_T^2> - < E_T>^2$ 

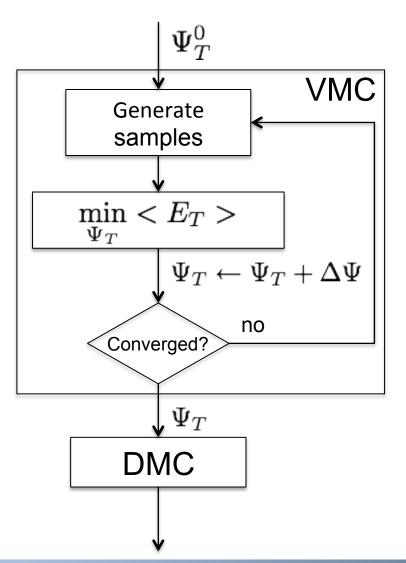
variance

- Minimize wall-clock time to reach a target error bar
- Efficiency of QMC simulations is high, when
  - Variance is small:  $\sigma \to 0 \; \text{as} \; \Psi_T \to \Psi \;\;\;\;$  (zero-variance) Physical insights & improved optimization
  - The rate of MC sample generation is high Parallelism, compact form of  $\Psi_T$  & optimized kernels





## HowTo for QMC Calculations



- Initial guess  $\Psi^0_T$ 
  - Compact, easy to evaluate, but close to true  $\Psi$

$$\Psi_T(\mathbf{R}) = J(\lbrace lpha \rbrace) \sum C_i D_i^{\uparrow}(\phi) D_i^{\downarrow}(\phi)$$

- Single-particle orbitals  $\{\phi\}$  e.g., KS or HF solution
- Find  $\{\alpha\}$  &  $\{C\}$  to optimize an object function: energy and variation minimization
- Projecting out the ground-state by applying a propagator  $e^{-\tau \hat{H}}$





### Diffusion Monte Carlo

```
for generation = 1 \cdots N_{MC} do
   for walker = 1 \cdots N_w do
       let \mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}
                                                                        Drift & Diffusion
      for particle i = 1 \cdots N do
          set \mathbf{r}_{i}' = \mathbf{r}_{i} + \delta
          let \mathbf{R}' = {\mathbf{r}_1 \dots \mathbf{r}_i' \dots \mathbf{r}_N}
          ratio \rho = \Psi_T(\mathbf{R}')/\Psi_T(\mathbf{R})
          if r \rightarrow r' is accepted then
              update state of a walker
          end if
       end for{particle}
       Compute E_L = \hat{H}\Psi_T(\mathbf{R})/\Psi_T(\mathbf{R})
       Reweight and branch walkers
                                                                                Branch
       Update E_T
   end for{walker}
end for{generation}
```





## Characteristics of QMC

```
DMC pseudo code
for generation = 1 \cdots N_{MC} do
   for walker = 1 \cdots N_w do
       let \mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}
       for particle i = 1 \cdots N do
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       Reweight and branch walkers
       Update E_T
   end for{walker}
end for{generation}
```

- Ample opportunity for parallelism
  - Configurations
  - K-point
  - Walker parallelization





## Characteristics of QMC

#### DMC pseudo code for generation = $1 \cdots N_{MC}$ do for walker = $1 \cdots N_w$ do let $\mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}$ for particle $i = 1 \cdots N$ do set $\mathbf{r}_{i}^{'} = \mathbf{r}_{i} + \delta$ let $\mathbf{R}' = {\mathbf{r}_1 \dots \mathbf{r}_i' \dots \mathbf{r}_N}$ ratio $ho = \Psi_T(\mathbf{R}')/\Psi_T(\mathbf{R})$ if $\mathbf{r} \to \mathbf{r}'$ is accepted then update state of a walker end if end for{particle} Compute $E_L = \hat{H}\Psi_T(\mathbf{R})/\Psi_T(\mathbf{R})$ Reweight and branch walkers Update $E_T$ end for{walker} end for{generation}

- Ample opportunity for parallelism
  - Configurations
  - K-point
  - Walker parallelization
- Freedom in  $\Psi_T$ 
  - Compute vs Memory
- Computationally demanding
  - Ratio, update & Local energy
  - Random access





# Characteristics of QMC

```
DMC pseudo code
for generation = 1 \cdots N_{MC} do
   for walker = 1 \cdots N_w do
       let \mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}
       for particle i = 1 \cdots N do
          set \mathbf{r}_{i}' = \mathbf{r}_{i} + \delta
          let \mathbf{R}' = {\mathbf{r}_1 \dots \mathbf{r}_i' \dots \mathbf{r}_N}
          ratio \rho = \Psi_T(\mathbf{R}')/\Psi_T(\mathbf{R})
          if r \rightarrow r' is accepted then
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          end if
       end for{particle}
       Compute E_L = \hat{H}\Psi_T(\mathbf{R})/\Psi_T(\mathbf{R})
       Reweight and branch walkers
       Update E_T
   end for{walker}
end for{generation}
```

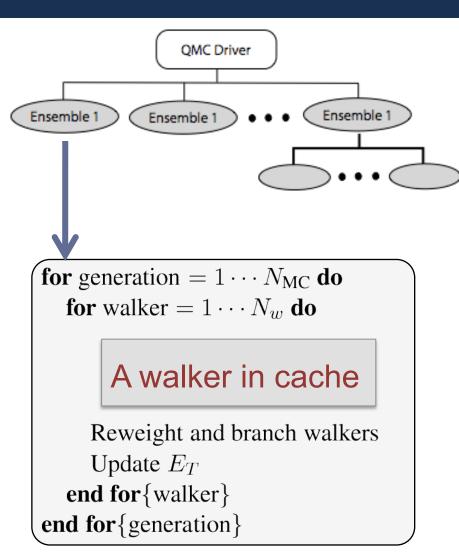
- Ample opportunity for parallelism
  - Configurations
  - K-point
  - Walker parallelization
- Freedom in  $\Psi_T$ 
  - Compute vs Memory
- Computationally demanding
  - Ratio, update & Local energy
  - Random access

- Communication light but need to
  - Global sum
  - Load balance





# Hierarchical Parallelization of QMC



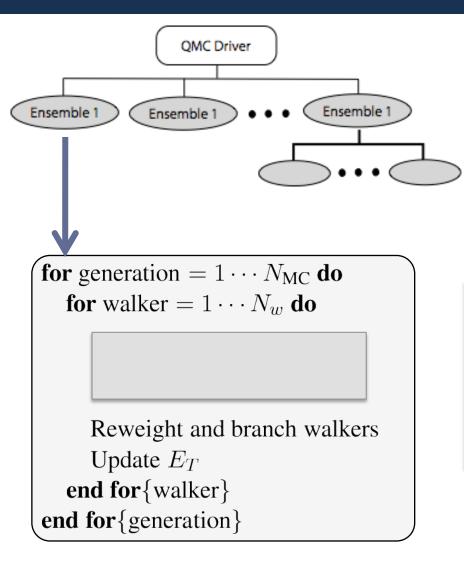
#### For a given N-electron system

- 1 Multiple instances of correlated configurations: any
- 2 Multiple k-points : 1-100 Critical to remove finite-size effects
- 3 Walker parallelization:

$$N_w=10^4-10^6$$
 Multi-core



# Hierarchical Parallelization of QMC



For a given N-electron system

- Multiple instances of correlated configurations: any
- 2 Multiple k-points : 1-100
  Critical to remove finite-size effects
- 3 Walker parallelization:

$$N_w = 10^4 - 10^6$$

4 N-particle:  $N-N^3$ 

**GPU** 

And, more parallelism can exposed

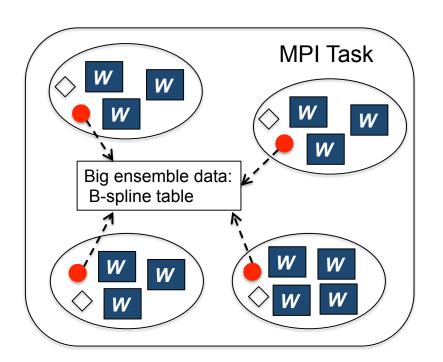
$$\Psi_T(\mathbf{R}) = \Pi_i \Psi_i, \hat{H} = \sum_i \hat{h}_i$$

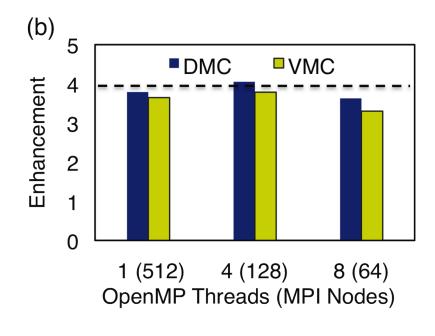




# Hybrid scheme on SMP

- Maximize performance and reduce the time-to-solution
  - MPI task per SMP, better per NUMA node
  - Multiple walkers per threads
  - Use all the hardware threads available



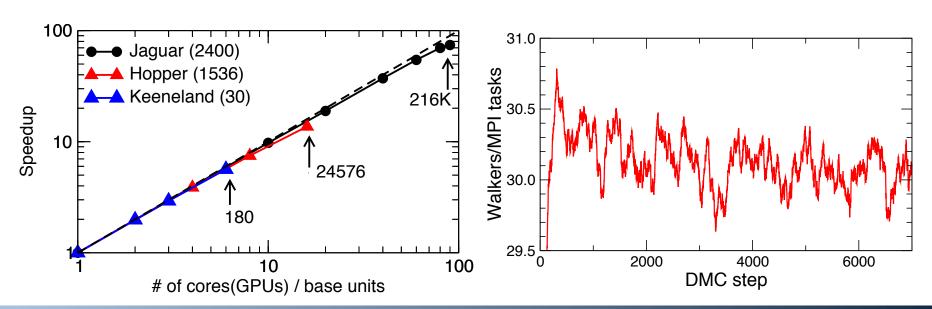






# Performance of Hybird QMC

- DMC scaling is almost perfect, > 90% efficiency
  - Limited by collectives for  $E_T, N_p^w < N^w >$
- Open/MPI hybrid helps more than memory footprint
  - Collectives scale O(P<sup>2</sup>) or O(P In P) for P tasks
  - Large average number of walkers per MPI task, thus small fluctuations: easy to balance walkers per node







## QMC on Clusters of SMPs

- Compute-heavy and communication-light nature makes QMC an easier parallel problem than other problems
- But, as the parallelism increases > 10<sup>4</sup>, many issues arise
  - Limited memory per core
  - MPI performance : collectives
  - I/O: initialization and checkpoint
- MPI/OpenMP provides QMC with simple but effective solutions
  - Standards of both commercial and HPC : rely on steady improvement of the HP infrastructure, compilers and libraries
  - Can exploit hierarchy of memory and communication
  - Large-shared memory per node : minimize data replications,
     while taking advantage of increasing hardware threads





## QMC on GPU

- Why GPU?
- Many threads, high floating-point performance, and bandwidth
- Tera- and peta-scale workstations
- A candidate for the future HPC architecture
- GPU port of QMCPACK\*
- Restructure the algorithm and data structure to exploit parallelism
- MPI for load balancing & reductions : high parallel efficiency

```
for walker = 1 \cdots N_w do
   let \mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}
   for particle i = 1 \cdots N do
       set \mathbf{r}_{i}' = \mathbf{r}_{i} + \delta
       let \mathbf{R}' = \{\mathbf{r}_1 \dots \mathbf{r}_i' \dots \mathbf{r}_N\}
       ratio \rho = \Psi_T(\mathbf{R}')/\Psi_T(\mathbf{R})
       if r \rightarrow r' is accepted then
           update state of a walker
       end if
   end for{particle}
   Compute E_L = \hat{H}\Psi_T(\mathbf{R})/\Psi_T(\mathbf{R})
   Reweight and branch walkers
   Update E_T
end for{walker}
```

<sup>\*</sup> Esler, Kim, Shulenburger&Ceperley, CISE (2010)

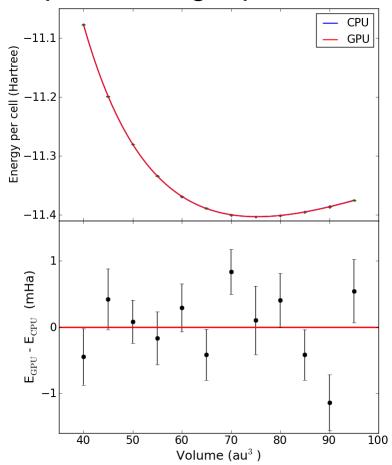




Loops

## QMC on GPU

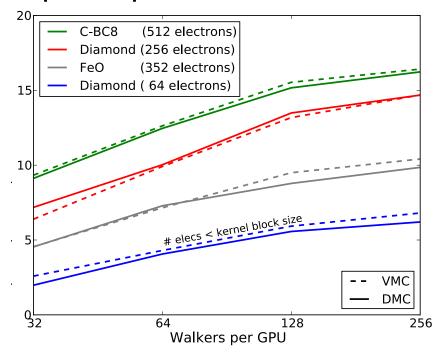
#### Impact of single precision



CPU: double

GPU: mixed, main kernels in single

#### Speedup: 1 GPU/ 4 cores



Performance data on NCSA Lincoln cluster

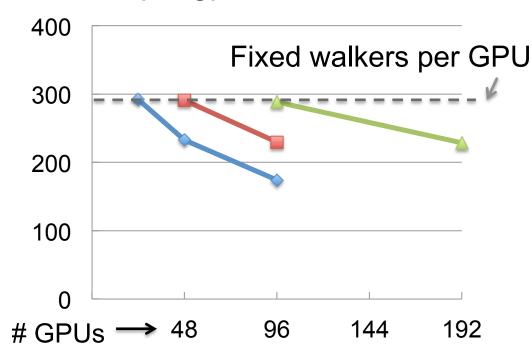
- nVidia G200 GPUs
- Intel Xeon (Harpertown)





# Scaling on multiple GPUs

#### MC sample/gpu/sec



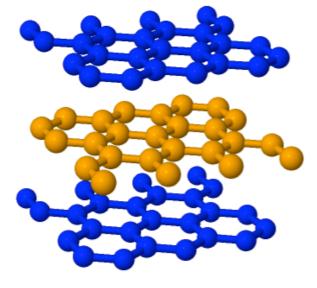
#### Target population





12288





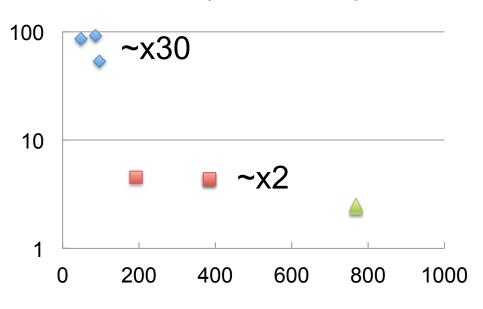
- 3x3x1 Graphite
  - 36 Carbon atoms
  - 144 electrons
- On Keeneland at NICS, each node has
  - Dual Hex-core X5560
  - 3 NVIDIA Fermi





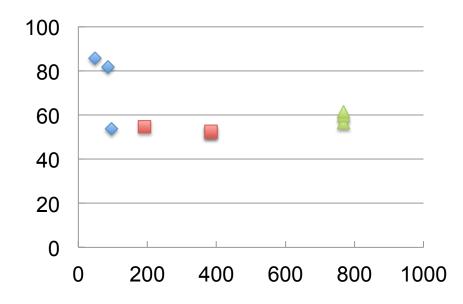
# Performance update

#### MC samples/(GPU,core)/sec



- NVIDIA Fermi (Keeneland)
- Intel Westmere (Keeneland)
- AMD MagnyCours (Hopper)

MC samples/(GPU, Node)/sec



MC samples/sec = figure of merit for QMC

\*4x4x1 graphite, 256 electrons





# Computational challenges for QMC

QMC positioned to harness the increasing computing powers of current and next generation of HPC

- Sufficient parallelism over walkers on current HPC systems
  - Petaflop multi-core systems
  - Teraflop GPU systems
- A lot of new sciences on petaflop heterogeneous systems, including Titan

## Reduce time per walker per DMC step: $O(N^2)$ - $O(N^3)$

- Fine-level parallelisms: light-weight threads, nested tasks
- Optimizations on multi-core chips: random-access of readonly data, private/shared cache reuse on NUMA systems
- Utilizing all the power of heterogeneous nodes





# Room for improvement

```
for generation = 1 \cdots N_{MC} do
                                                                    \Psi_T(\mathbf{R}) = \Pi_k \Psi_k
   for walker = 1 \cdots N_w do
      let \mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}
                                                                       \hat{H} = \sum_{k} \hat{h}_{k}
      for particle i = 1 \cdots N do
                                              node
         set \mathbf{r}_{i}' = \mathbf{r}_{i} + \delta
                                                             T Psi<T>::ratio(int i)
         let \mathbf{R}' = {\mathbf{r}_1 \dots \mathbf{r}_i' \dots \mathbf{r}_N}
         ratio \rho = \Psi_T(\mathbf{R}')/\Psi_T(\mathbf{R})
                                                                T r(1.0);
                                                                for (int k=0; k<Z.size(); ++k)</pre>
         if r \rightarrow r' is accepted then
                                                                   r *= Z[k]->ratio(P,i);
            update state of a walker
                                                                return r;
         end if
      end for{particle}
      Compute E_L = H\Psi_T(\mathbf{R})/\Psi_T(\mathbf{R})
                                                               Hamiltonian<T>::evaluate()
      Reweight and branch walkers
                                                               T eloc=0.0;
      Update E_T
                                                                for(int k=0; k<H.size(); ++k)</pre>
                                                                  eloc += H[k]->evaluate(P);
   end for{walker}
                                                                return eloc
end for{generation}
```





# **Core Computations**

For each walker,

```
let \mathbf{R} = \{\mathbf{r}_1 \dots \mathbf{r}_N\}

for particle i = 1 \dots N do

set \mathbf{r}_i' = \mathbf{r}_i + \delta

let \mathbf{R}' = \{\mathbf{r}_1 \dots \mathbf{r}_i' \dots \mathbf{r}_N\}

ratio \rho = \Psi_T(\mathbf{R}')/\Psi_T(\mathbf{R})

if \mathbf{r} \to \mathbf{r}' is accepted then

update state of a walker

end if

end for{particle}

Compute E_L = \hat{H}\Psi_T(\mathbf{R})/\Psi_T(\mathbf{R})
```

All about  $\Psi_T$ 

$$\delta = r + au 
abla_i \ln \Psi_T$$
 Quantum force  $rac{\Psi_T(\mathbf{r}_1 \cdots \mathbf{r}_i^{'} \cdots \mathbf{r}_N)}{\Psi_T(\mathbf{r}_1 \cdots \mathbf{r}_i \cdots \mathbf{r}_N)}$   $\Psi_T \leftarrow \Psi_T(\mathbf{r}_1 \cdots \mathbf{r}_i^{'} \cdots \mathbf{r}_N)$   $f(\{\mathbf{R}\}, \nabla \ln \Psi_T, \nabla^2 \ln \Psi_T)$ 

Use 
$$\Psi_T = \Pi_i \Psi_i \implies \ln \Psi_T = \sum_i \ln \Psi_i$$





## Slater-Jastrow for Electrons

$$\Psi_T(\mathbf{R}) = e^{J_1 + J_2 + \cdots} \sum C_i D_i^{\uparrow}(\phi) D_i^{\downarrow}(\phi) \qquad N = N^{\uparrow} + N^{\downarrow}$$

Correlation (Jastrow)

$$J_1 = \sum_i^N \sum_I^{ions} u_1(|\mathbf{r}_i - \mathbf{r}_I|)$$

$$J_2 = \sum_{i 
eq j}^N u_2(|\mathbf{r}_i - \mathbf{r}_j|)$$

**Anti-symmetric function** (Pauli principle)

$$D_i^\uparrow = \det \begin{vmatrix} \phi_1(\mathbf{r}_1) & \cdots & \phi_1(\mathbf{r}_{N^\uparrow}) \\ \vdots & \vdots & \vdots \\ \phi_{N^\uparrow}(\mathbf{r}_1) & \cdots & \phi_{N^\uparrow}(\mathbf{r}_{N^\uparrow}) \end{vmatrix}$$
 Single-particle orbitals

- Computational complexity per MC step
  - Evaluation  $\{\phi\}$
  - **Determinant evaluation**
  - **Jastrow evaluation**

$$\mathcal{O}(N^2N_{spo})$$

$$\mathcal{O}(N^3)$$

$$\mathcal{O}(N) - \mathcal{O}(N^3)$$





# Single-particle orbitals

• Linear combinations of basis functions  $N_{spo} \propto N_b Op(\Phi)$ 

$$\phi_i = \sum_k^{k=N_b} c_k^i \Phi_k$$

- Typically the solutions of simpler theories, i.e.  $C's \& \{\Phi\}$  from Hartree-Fock or DFT calculations
- SPO can take various forms

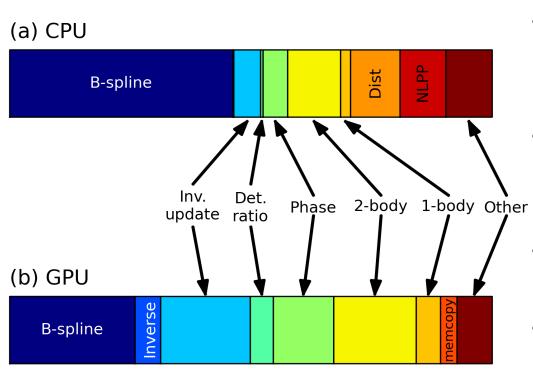
SPO Type	$N_b$	$Op(\Phi)$	Memory Use
Molecular orbitals	$\mathcal{O}(N)$	Medium-High	Low
Plane waves	$\mathcal{O}(N)$	High	Medium
B-spline	Fixed	Low	High

Best solution for large-scale QMC on SMPs





# Analysis on current CPU & GPU



Breakup of compute kernels

- QMCPACK achieves high efficiency by amortizing threads & memory
- As the system size and complexity grows, each kernel takes longer
- Can afford overhead for task-based parallelism
- But, difficult to balance the load among tasks: device and problem dependent





# Strategy to further accelerate QMC

- Task-based parallelism with smart allocators on heterogeneous nodes
- Exploit generic programming
  - Specialization on devices: allocators, containers, algorithms
  - Hide low-level programming but optimize the kernels with the best option(s) available
  - Auto-tuning of SIMD kernels
- Stick to standards: C++, OpenMP, Pthreads and MPI
  - Heavy lifting by the compilers
  - Vendor optimized communication and numerical libraries
- Cope with the changes





#### Conclusions

- QMC has kept up with the HPC evolution and will continue improving predictive powers in physics, materials and chemistry
  - ✓ Clusters of multi- and many-core SMP
  - ✓ Clusters of GPU
  - Clusters of hybrid
  - What is next
- More to be done improve science productivity
  - Reduce impacts of application-level, software and hardware faults: Algorithms for robust and fault-tolerant simulations
  - Faster off-node communication and I/O





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